Ring vortices generated electromagnetically

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If an electric current of uniform density j_0 is passed axially through a stationary fluid between concentric cylinders of radii r_1 and r_2 ($> r_1$), the fluid is stable to axisymmetric disturbances only if the damping provided by viscosity and electrical resistivity is sufficiently large. It is shown herein that the fluid may also be stabilized by passing a line current J along the axis, sufficient conditions for stability being

$$J \leqslant -\pi j_0(r_2^2-r_1^2), \quad \text{or} \quad \geqslant J\ \pi j_0 r_1^2.$$

The values of J needed to stabilize the fluid when the fluid has non-zero viscosity and finite conductivity are calculated for the case $r_2 - r_1 \ll r_1$. In this latter case, the ring vortices which exist under conditions of neutral stability are exactly the same as those for flow between rotating cylinders if J and j_0 have the same sign, and if J is not very small compared with $\pi j_0 r^2$.

1. Introduction

In a remarkable paper (Taylor 1923), Taylor presented the results of his analytical and experimental investigation of the stability of a viscous fluid between two rotating cylinders. The vortices which he found in the fluid for unstable conditions have since been found to be present in many fluid flows with curved streamlines. The cause of these vortices is here dynamical in nature, i.e. it is due to the centripetal acceleration of the fluid which is tantamount to a centrifugal force. Can similar ring vortices be created in a quiescent fluid? The answer is in the affirmative if electromagnetic forces are allowed. In the following sections, it will be shown that Taylor vortices can occur in a fluid between concentric cylinders if a longitudinal electric current passes along the axis of the cylinders and another passes through the fluid. Specific results are given for small differences in radii. The cause of instability is the *centripetal* electromagnetic body force acting on the fluid in the undisturbed state.

2. The undisturbed state

The relationship between magnetic field strength $\mathbf{H}$ and current density $\mathbf{j}$ is

$$\operatorname{curl} \mathbf{H} = 4\pi \mathbf{j}. \tag{1}$$

In cylindrical co-ordinates (r, θ, z), a current J along the centre line (z-axis) of the cylinders will give rise to a circular magnetic field equal to $2J/r$, current being counted as positive if it is in the positive z-direction. The circular magnetic field due to a current of density j_0 passing between the cylinders in the axial direction is $2\pi j_0(r^2-r_1^2)/r$, if the radii of the cylinders are denoted by r_1 and $r_2\,(>r_1)$. Thus the undisturbed state is characterized by

$$\bar{H}_r = \bar{H}_z = 0, \quad \bar{H}_\theta = 2\pi\left(\frac{J'}{\pi r}+j_0 r\right), \quad J' = J-\pi j_0 r_1^2. \tag{2}$$

It can be shown that a quiescent state of the fluid is consistent with the magnetic field given by equations (2), and that the only effect the field has is to change the (hydrostatic) pressure by an amount consistent with the equation of equilibrium.

The electromagnetic body force per unit volume of the fluid is $-\mu j_0 H_\theta$ acting in the (outward) radial direction. Since this body force is similar to the pseudo-body-force (so-called centrifugal force) in the case of a fluid under rotation, instability of the fluid can be expected to occur under suitable conditions.

3. Formulation of the problem

With (u, v, w) and (H_r, H_θ, H_z) denoting the components of the velocity and of the magnetic field strength in the directions of the (r, θ, z)-co-ordinate lines, the equations of motion for an incompressible fluid are

$$\rho\left(\frac{Du}{D\tau}-\frac{v^2}{r}\right) = -\frac{\partial\chi}{\partial r}+\rho\nu\left(\nabla^2 u-\frac{u}{r^2}-\frac{2}{r^2}\frac{\partial v}{\partial\theta}\right)+\frac{\mu}{4\pi}\left(\frac{\mathscr{D}H_r}{\mathscr{D}\tau}-\frac{H_\theta^2}{r}\right),$$

$$\rho\left(\frac{Dv}{D\tau}+\frac{uv}{r}\right) = -\frac{1}{r}\frac{\partial\chi}{\partial\theta}+\rho\nu\left(\nabla^2 v-\frac{v}{r^2}+\frac{2}{r^2}\frac{\partial u}{\partial\theta}\right)+\frac{\mu}{4\pi}\left(\frac{\mathscr{D}H_\theta}{\mathscr{D}\tau}+\frac{H_r H_\theta}{r}\right),$$

$$\rho\frac{Dw}{D\tau} = -\frac{\partial\chi}{\partial z}+\rho\nu\nabla^2 w+\frac{\mu}{4\pi}\frac{\mathscr{D}H_z}{\mathscr{D}\tau},$$

in which

$$\chi = p+\frac{\mu|\mathbf{H}|^2}{8\pi}+\rho\Omega,$$

$$\frac{\mathscr{D}}{\mathscr{D}\tau} \equiv H_r\frac{\partial}{\partial r}+\frac{H_\theta}{r}\frac{\partial}{\partial\theta}+H_z\frac{\partial}{\partial z},$$

$$\frac{D}{D\tau} \equiv \frac{\partial}{\partial\tau}+u\frac{\partial}{\partial x}+\frac{v}{r}\frac{\partial}{\partial\theta}+w\frac{\partial}{\partial z},$$

$$\nabla^2 \equiv \frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}+\frac{\partial^2}{\partial z^2},$$

and ρ is the density, τ the time, ν the kinematic viscosity, μ the magnetic permeability, and Ω the gravitational potential per unit mass. The equations for the magnetic field are

$$\frac{DH_r}{D\tau} = \frac{\mathscr{D}u}{\mathscr{D}\tau}+\eta\left(\nabla^2 H_r-\frac{H_r}{r^2}-\frac{2}{r^2}\frac{\partial H_\theta}{\partial\theta}\right),$$

$$\frac{DH_\theta}{D\tau}+\frac{vH_r}{r} = \frac{\mathscr{D}v}{\mathscr{D}\tau}+\frac{H_\theta u}{r}+\eta\left(\nabla^2 H_\theta-\frac{H_\theta}{r^2}+\frac{2}{r^2}\frac{\partial H_r}{\partial\theta}\right),$$

$$\frac{DH_z}{D\tau} = \frac{\mathscr{D}w}{\mathscr{D}\tau}+\eta\nabla^2 H_z,$$

in which η is the magnetic diffusivity.

A disturbance of the stationary fluid will give rise to small velocity components (u, v, w), and a deviation from the equilibrium magnetic field denoted by (h_r, h_θ, h_z). The total magnetic field is then given by

$$h_r, \quad 2\pi\left(\frac{J'}{\pi r}+j_0 r\right)+h_\theta, \quad h_z.$$

If these are substituted in the equations of motion and the equations of magnetic diffusion and all quadratic terms in u, v, w, and the h's are neglected, and if axisymmetry is assumed, the following linearized equations are obtained:

$$\rho\frac{\partial u}{\partial \tau} = -\frac{\partial \chi'}{\partial r}+\rho\nu\left(\nabla^2 u-\frac{u}{r^2}\right)-\mu\left(\frac{J'}{\pi r^2}+j_0\right)h_\theta, \tag{3}$$

$$\rho\frac{\partial v}{\partial \tau} = \rho\nu\left(\nabla^2 v-\frac{v}{r^2}\right)+\mu j_0 h_r, \tag{4}$$

$$\rho\frac{\partial w}{\partial \tau} = -\frac{\partial \chi'}{\partial z}+\rho\nu\nabla^2 w, \tag{5}$$

$$\frac{\partial h_r}{\partial \tau} = \eta\left(\nabla^2 h_r-\frac{h_r}{r^2}\right), \tag{6}$$

$$\frac{\partial h_\theta}{\partial \tau} = \frac{4J'}{r^2}u+\eta\left(\nabla^2 h_\theta-\frac{h_\theta}{r^2}\right), \tag{7}$$

$$\frac{\partial h_z}{\partial \tau} = \eta\nabla^2 h_z. \tag{8}$$

in which $$\nabla^2 \equiv \frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}, \quad \chi' = p'+\frac{\mu}{4\pi}\bar{H}_\theta h_\theta,$$

with p' denoting the pressure perturbation. The equation of continuity is

$$\frac{\partial(ru)}{\partial r}+\frac{\partial(rw)}{\partial z} = 0, \tag{9}$$

since compressibility can be neglected.

From the form of equations (6) and (8) we can conclude that h_r and h_z will be damped out if they are not initially everywhere zero (see Yih 1959). Then from equation (4) we conclude further that v will also be damped out. The differential equations to be dealt with are then equations (3), (5) and (7). Eliminating χ' from equations (3) and (5), we have

$$\rho\frac{\partial}{\partial \tau}\left(\frac{\partial w}{\partial r}-\frac{\partial u}{\partial z}\right) = \rho\nu\left(\nabla^2-\frac{1}{r^2}\right)\left(\frac{\partial w}{\partial r}-\frac{\partial u}{\partial z}\right)+\mu\left(\frac{J'}{\pi r^2}+j_0\right)\frac{\partial h_\theta}{\partial z}. \tag{10}$$

We define dimensionless variables

$$(r', z') = \left(\frac{r}{r_1}, \frac{z}{r_1}\right), \quad t = \frac{\tau\nu}{r_1^2},$$

$$(u_1, w_1) = \left(\frac{ur_1}{\nu}, \frac{wr_1}{\nu}\right), \quad h_1 = \frac{h_\theta}{j_0 r_1},$$

where r_1 and r_2 are the radii of the inner and outer cylinders. Then, on dropping the primes on r and z, we can write equations (10) and (7) as

$$\frac{\partial}{\partial t}\left(\frac{\partial w_1}{\partial r}-\frac{\partial u_1}{\partial z}\right) = \left(\nabla^2-\frac{1}{r^2}\right)\left(\frac{\partial w_1}{\partial r}-\frac{\partial u_1}{\partial z}\right)+A\left(\frac{B}{r^2}+1\right)\frac{\partial h_1}{\partial z}, \tag{11}$$

$$\frac{\partial h_1}{\partial t} = \frac{\eta}{\nu}\left(\nabla^2-\frac{1}{r^2}\right)h_1+\frac{4\pi B}{r^2}u_1, \tag{12}$$

in which

$$A = \frac{\mu j_0^2 r_1^4}{\rho\nu^2}, \quad B = \frac{J'}{\pi j_0 r_1^2}. \tag{13}$$

Following Taylor, we can make the following substitutions:

$$(u_1, w_1, h_1) = [U(r)\cos mz, \quad W(r)\sin mz, \quad h(r)\cos mz]\, e^{\sigma t},$$

in which m is the wave number for the z-direction. The equation of continuity then becomes

$$rU'+U+mrW = 0, \tag{14}$$

and equations (11) and (12) become

$$(L-m^2-\sigma)(L-m^2)\,U = -m^2A\left(\frac{B}{r^2}+1\right)h, \tag{15}$$

$$\left(L-m^2-\frac{\sigma\nu}{\eta}\right)h = -\frac{4\pi B\nu}{\eta r^2}U, \tag{16}$$

in which

$$L = D\left\{\frac{1}{r}D(r\ )\right\}, \quad D = \frac{d}{dr}.$$

With

$$U = -m^2Af \quad \text{and} \quad N = \frac{4\pi AB\nu}{\eta},$$

equations (15) and (16) can be written as

$$(L-m^2-\sigma)(L-m^2)f = \left(\frac{B}{r^2}+1\right)h, \tag{17}$$

$$\left(L-m^2-\frac{\sigma\nu}{\eta}\right)h = m^2N\frac{f}{r^2}. \tag{18}$$

The boundary conditions corresponding to

$$U = W = 0 \quad \text{at} \quad r = 1 \quad \text{and} \quad r = \frac{r_2}{r_1} = \alpha$$

are

$$f = Df = 0 \quad \text{at} \quad r = 1 \text{ and } r = \alpha. \tag{19}$$

The simplest realistic boundary condition for h is that which corresponds to zero electrical conductivity of the walls, that is, to $j_r = 0$. But

$$j_r = \frac{1}{r}\frac{\partial h_3}{\partial\theta}-\frac{\partial h_\theta}{\partial z} = -\frac{\partial h_\theta}{\partial z} \quad \text{(for axisymmetry)},$$

and so the boundary condition for h is

$$h = 0 \quad \text{at} \quad r = 1 \text{ and } r = \alpha \tag{20}$$

for non-conducting walls. The task is to find the relationship between N and m for a given value of B, from the differential system consisting of equations (17) and (20).

4. Sufficient condition for stability

A sufficient condition for stability can be given on physical grounds, in the manner of Rayleigh (1916), or on mathematical grounds, in the manner of Synge (1938). The physical proof relies upon the fact that for a magnetically non-diffusive fluid ($\eta = 0$) the lines of force move with the fluid. The proof for this well-known fact is identical with the proof for vorticity lines in an inviscid fluid (Lamb 1945, p. 204), and is available elsewhere. In § 3 of this paper it has been shown that H_r and H_z (or h_r and h_z) will be damped out. In a discussion of the sufficient condition for stability it can thus be assumed that only H_θ is different from zero. Since the total magnetic flux round a thin material ring of fluid is constant, and since by continuity the volume of this ring must be constant, so that its cross-section varies inversely as its radius r', H_θ must be equal to kr' (r' dimensional), with k as the constant (for the ring as it moves) of proportionality. The body force per unit volume in the r-direction, which is in general (Yih 1959)

$$\frac{\mu}{4\pi}\left(-\frac{\frac{1}{2}(\partial|\mathbf{H}|^2)}{\partial r}+H_r\frac{\partial H_r}{\partial r}+\frac{H_\theta}{r}\frac{\partial H_r}{\partial \theta}+H_z\frac{\partial H_r}{\partial z}-\frac{H_\theta^2}{r}\right),$$

can in the present discussion be written as

$$-\frac{\mu}{4\pi}H_\theta\left(\frac{\partial H_\theta}{\partial r}+\frac{H_\theta}{r}\right).$$

Now imagine the thin material ring with a magnetic flux in the θ-direction to be instantaneously situated at the position indicated by r'. The sole effect of the term

$$-\frac{\mu}{4\pi}H_\theta\frac{\partial H_\theta}{\partial r}$$

is to reduce the pressure throughout the ring by an amount $\mu H_\theta^2/8\pi$, and therefore has no effect on the work done by the ring against pressure applied externally on the surface of the ring. Consequently, the only force acting on the ring which can be properly counted as a body force is

$$-\frac{\mu}{4\pi}\frac{H_\theta^2}{r}.$$

Since the instantaneous value for r is r' for the thin ring, and since $H_\theta = kr'$, this force can be written as

$$-\frac{\mu}{4\pi}k^2r'.$$

The potential energy due to this centripetal force is then $\mu k^2r'^2/8\pi$, which increases with r' for any particular value of k.

Returning to the distribution of the mean field $\bar{H}_\theta$ specified earlier in this paper, we can imagine the fluid in its mean configuration to be composed of thin shells, each with a different value for k given by

$$k = \frac{\bar{H}_\theta}{r} = 2\pi\left(\frac{J'}{\pi r^2}+j_0\right).$$

Since a higher value of k^2 corresponds to a 'heavier' fluid, and since stability will ensue if a 'heavier' fluid shell occupies a position of lower potential energy (hence smaller r), one concludes that a sufficient condition for stability is that the quantity

$$\left(\frac{J'}{\pi r^2}+j_0\right)^2$$

does not increase outwards—a situation which is possible only if

$$J \leqslant -\pi j_0(r_2^2-r_1^2) \quad \text{or} \quad J \geqslant j_0 r_1^2.$$

In this physical proof magnetic diffusivity has been neglected. It is tacitly assumed that, if a fluid with no magnetic diffusivity is stable, one with magnetic diffusivity will be so *a fortiori*. For the type of instability under discussion, this is a valid assumption, as will be demonstrated by a mathematical proof (of the sufficient conditions just reached), in which the effects of viscosity and magnetic diffusivity are not neglected. Whether magnetic diffusivity can have a destabilizing effect on certain flows, in the manner that viscosity can sometimes be destabilizing, is not known. An investigation of this possibility would be highly interesting.

For a mathematical proof of the same result one turns to the dimensionless equations (17) to (20). Multiplying equation (17) by $r\bar{f}$ ($\bar{f}$ being the complex conjugate of f) and integrating (by parts if necessary) with respect to r between 1 and α, we have, upon utilization of the boundary conditions on f,

$$I_2+(2m^2+\sigma)\,I_1+m^2(m^2+\sigma)\,I_0 = \int_1^\alpha \left(\frac{B}{r^2}+1\right) rh\bar{f}\,dr, \tag{21}$$

in which

$$I_0 = \int_1^\alpha r\,|f|^2\,dr, \quad I_1 = \int_1^\alpha \frac{1}{r}\,|Drf|^2\,dr, \quad I_2 = \int_1^\alpha r|Lf|^2\,dr.$$

Similarly, by multiplying equation (18) by $r\bar{h}$ and $r^3\bar{h}$ and integrating, one obtains, respectively,

$$H_1+\left(m^2+\frac{\nu\sigma}{\eta}\right)H_0 = -m^2N\int_1^\alpha \frac{f\bar{h}}{r}\,dr, \tag{22}$$

$$H_3+\left(m^2+\frac{\nu\sigma}{\eta}\right)H_2+2\int_1^\alpha r\bar{h}D(rh)\,dr = -m^2N\int_1^\alpha rf\bar{h}\,dr, \tag{23}$$

in which

$$H_0 = \int_1^\alpha r\,|h|^2\,dr, \quad H_1 = \int_1^\alpha \frac{1}{r}\,|Drh|^2\,dr, \quad H_2 = \int_1^\alpha r^3\,|h|^2\,dr, \quad H_3 = \int_1^\alpha r\,|Drh|^2\,dr.$$

Equations (21) to (23) can be suitably combined to eliminate the integrals on the right-hand sides. The result is

$$m^2N(I_2+2m^2I_1+m^4I_0)+B(H_1+m^2H_0)+(H_3+m^2H_2)$$
$$-2\int_1^\alpha r\bar{h}D(rh)\,dr+\sigma\left(m^2NI_1+m^4NI_0+\frac{B\nu}{\eta}H_0+\frac{\nu}{\eta}H_2\right) = 0. \tag{24}$$

But since
$$\int_1^\alpha r\bar{h}D(rh)\,dr+\int_1^\alpha rhD(r\bar{h})\,dr = r^2h\bar{h}\,|_1^\alpha = 0,$$

the real part of the integral in equation (24) is zero. Taking the real part of (24), we have

$$m^2N(I_2+2m^2I_1+m^4I_0)+B(H_1+m^2H_0)+(H_3+m^2H_2)$$
$$+\sigma_r\left(m^2NI_1+m^4NI_0+\frac{B\nu}{\eta}H_0+\frac{\nu}{\eta}H_2\right)=0. \quad (25)$$

Now from the definitions of the integrals denoted by H, it is evident that

$$\alpha^2H_0 > H_2, \quad \text{and} \quad \alpha^2H_1 > H_3.$$

Thus, since N and B are of the same sign, if

$$-B \geqslant \alpha^2, \quad \text{or} \quad B > 0, \quad (26)$$

it follows from equation (25) that σ_r is negative, and the fluid is stable. With the definition of B given by (13), the sufficient condition of stability is therefore again found to be

$$J \leqslant -\pi j_0(r_2^2-r_1^2) \quad \text{or} \quad J \geqslant \pi j_0 r_1^2. \quad (27)$$

5. Solution for small spacings

For small spacings of the cylinders, the operator L in (17) and (18) can be replaced by D^2. If the dimensionless parameters are now re-defined as

$$t=\frac{\tau\nu}{d^2}, \quad \xi=\frac{r-r_1}{d}\,(r \text{ dimensional}), \quad m=\frac{2\pi d}{\lambda},$$

in which d is r_2-r_1, and λ is the wavelength in the z-direction, equations (17) and (18) can be replaced by

$$(D^2-m^2-\sigma)(D^2-m^2)f=\left(\frac{d}{r_1}\right)^4\left[(B+1)-\frac{2Bd}{r_1}\xi\right]h, \quad (28)$$

$$\left(D^2-m^2-\frac{\nu\sigma}{\eta}\right)h=m^2N\left(1-\frac{2d}{r_1}\xi\right)f, \quad (29)$$

where D stands now for $d/d\xi$. The boundary conditions are

$$f=Df=0 \quad \text{at} \quad \xi=0 \text{ and } 1, \quad (30)$$

$$h=0 \quad \text{at} \quad \xi=0 \text{ and } 1. \quad (31)$$

We shall investigate the stability for the cases in which the two currents are in the same direction ($-B<1$), so that the electromagnetic body force on the undisturbed state is centripetal. If B is not nearly equal to -1, equations (28) and (29) become (since d/r_1 is assumed to be very small)

$$(D^2-m^2-\sigma)(D^2-m^2)f=(B+1)\left(\frac{d}{r_1}\right)^4h, \quad (32)$$

$$\left(D^2-m^2-\frac{\nu\sigma}{\eta}\right)h=m^2Nf. \quad (33)$$

Effectively the same equations with the same boundary conditions as for the problem at hand have been solved exactly by Pellew & Southwell (1940), who also proved that for neutral stability σ is zero and not purely imaginary. Comparing equations (32) and (33) with Pellew & Southwell's equations (see Lin 1955, p. 108), we find that the parameter corresponding to Pellew & Southwell's R (Rayleigh number) is

$$T \equiv -(1+B)\,N\left(\frac{d}{r_1}\right)^4 = \frac{\mu J(\pi j_0 r_1^2 - J)}{\pi\rho\nu\eta}\left(\frac{d}{r_1}\right)^4.$$

According to Pellew & Southwell's solution, then,

$$T = 1707{\cdot}8. \tag{34}$$

If $1+B$ is positive but of the same magnitude as d/r_1, and if the principle of exchange of stabilities is assumed, equations (28) and (29) become

$$(D^2-m^2)^2 f = (1+B)\left(\frac{d}{r_1}\right)^4 (1+\beta\xi)\,h, \tag{35}$$

$$(D^2-m^2)\,h = m^2 N f. \tag{36}$$

where

$$\beta = -\frac{2dB}{r_1(1+B)}. \tag{37}$$

The boundary conditions are still specified by (30) and (31). Solution of the differential system for three values of β by the method of Chandrasekhar (1954) yields the corresponding critical values of T as given in table 1.

β ...	0·25		0·5		1·0	
m ...	3·12	3·13	3·12	3·13	3·12	3·13
T (1st approx.)	1524·5	1524·6	1372·1	1372·1	1143·4	1143·4
T (2nd approx.)	1524·4	1524·4	1371·6	1371·7	1142·4	1142·4
T (3rd approx.)	1518·0	1518·0	1365·9	1366·0	1137·7	1137·7

TABLE 1

Computation for the case of negative $J(1+B < 0)$ has not been performed. However, from the definition of N it can be seen that N is negative for negative J, and from the forms of (35) and (36) it can be seen that the fluid is probably stable for negative J unless $|1+B|$ is very small and β large. For very small $|1+B|$, the modified parameter

$$T' = \beta T$$

can be advantageously used instead of T. A table for T' can be easily constructed from table 1. The value of T' for negative J of small magnitude or for small $|1+B|$ is not much different from that for small positive J, so that a rough measure of it can be obtained by extrapolation from the table for T'. This measure can be improved by extending the range of β in Table 1, and hence in the T'-table derived therefrom.

6. Feasibility of experiment

For an experiment the most favourable value of B for the occurrence of ring vortices is of course $-1/2$. If d/r_1 is 0·1, for the critical case

$$\frac{\mu}{4}\frac{\pi j_0^2 r_1^4}{\rho\nu\eta} = 1707{\cdot}8\times 10^4.$$

For mercury, we have, in e.m.u.,

$$\eta = \frac{1}{4\pi\mu\sigma} = 8000\,\mathrm{cm^2/sec}, \quad \mu = 1\,\mathrm{e.m.u.},$$

$$\nu = 1{\cdot}12 \times 10^{-3}\,\mathrm{cm^2/sec}, \quad \rho = 13{\cdot}6\,g/\mathrm{c.c.}$$

For r_1 equal to 10 cm and d equal to 1 cm,

$$j_0 = 257\,\mathrm{e.m.u.} = 2570\,\mathrm{amp/cm^2}.$$

The total current through the annular space is then

$$20\pi j_0 = 1{\cdot}62 \times 10^5\,\mathrm{amp},$$

which is obviously a tremendous current. However, this current is the same (for the same d/r_1) regardless of the size of the apparatus, and the total power per metre (which is roughly 4160 kW for $r = 10$ cm) decreases in inverse proportion to the square of the lateral dimension of the apparatus. Furthermore, the heat capacity of mercury per metre of length increases in direct proportion to the square of the lateral dimension. Thus the danger of boiling decreases rapidly as the lateral size increases. For highly ionized gases with a density much smaller than that of mercury and with very high conductivity, the necessary current for ring vortices to occur will be very much reduced. Unfortunately a simple experiment with a small apparatus does not appear feasible.

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